CSCI 202 Research Methods

Exploring and Experimenting with Simulation

L. FELIPE PERRONE

A static model

Predicting profits for furniture sales

Simulation Model for Special Promotion Furniture Sale				fixed by contract
				input data
Stock ordered (S):	3000			calculated data
Unit cost for stock (C):	$175.00			
			Distribution Parameters	
			Lower	Upper
Demand within first 8 weeks (V):	2667		500	3500
Sales within first 8 weeks (V):	2667			
Initial price (R):	$251		200	300
Sales after first 8 weeks (S-V):	333			
Discount (D):	0.2			
Sale price (R*D):	0.5			
Profit (P):	$144,343			
Note: Google Sheets refresh on browser reload command				

Time has no bearing on this model

A static model

Predicting profits for furniture sales

This one is on Moodle:

Spreadsheet simulation

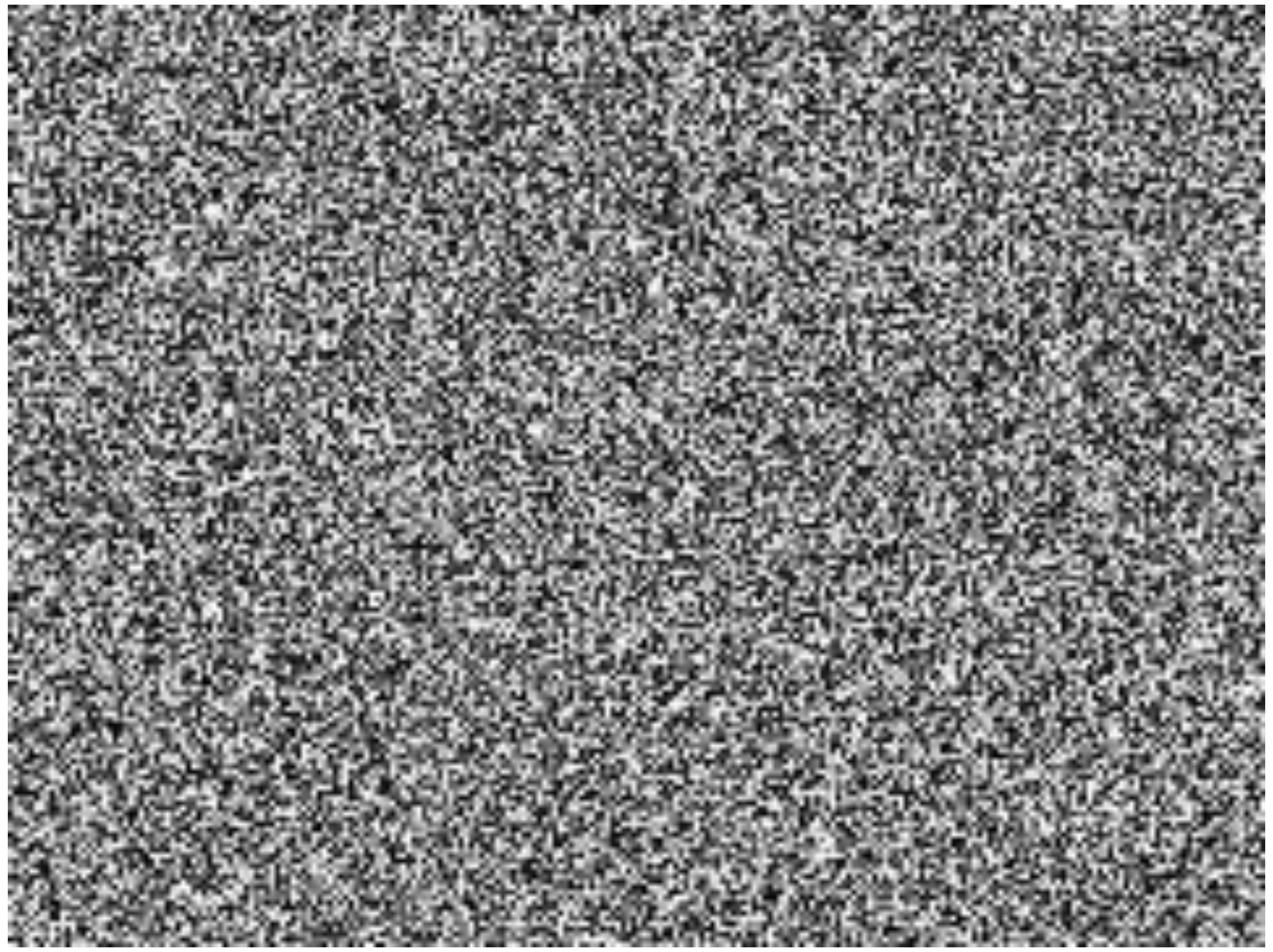
What is …

- a *random number generator*?
- a *pseudo-random number generator*?

Why should we need randomness?

"Real" *random numbers*

There is entropy in nature. If you can identify a source of entropy, you can "mine" random numbers from it.



white noise

Computers are deterministic…

If computers are fully deterministic, you need to do some work to get them to give you random numbers…

In Linux, look to **`/dev/random`** for random numbers. (You cannot "control" them.)

Pseudo-random number generators (PRNGs)

COMPUTING PRACTICES

Edgar H. Sibley
Panel Editor

Practical and theoretical issues are presented concerning the design, implementation, and use of a good, minimal standard random number generator that will port to virtually all systems.

RANDOM NUMBER GENERATORS: GOOD ONES ARE HARD TO FIND

STEPHEN K. PARK AND KEITH W. MILLER

An important utility that digital computer systems should provide is the ability to generate random numbers. Certainly this is true in scientific computing where many years of experience has demonstrated the importance of access to a good random number generator. And in a wider sense, largely due to the encyclopedic efforts of Donald Knuth [18], there is now a realization that random number generation is a concept of fundamental importance in many different areas of computer science. Despite that, the widespread adoption of good, portable, *industry standard* software for considerations developed over a period of several years while teaching a graduate level course in simulation. Students taking this course work on a variety of systems and their choices typically run the gamut from personal computers to mainframes. With Knuth's advice in mind, one important objective of this course is for all students to write and use implementations of a good, minimal standard random number generator that will port to *all* systems. For reasons discussed later, this minimal standard is a multiplicative linear congruential generator [18, p. 10] with multiplier 16807 and

WHAT MAKES A PRNG "GOOD" ?

Pseudo-random number generator

$$Z_i = (aZ_{i-1} + c) \ \% \ m$$

LINEAR CONGRUENTIAL GENERATOR

LEHMER GENERATOR

Z_0 = seed

a, c, and m = carefully chosen constants

$$\{\underbrace{Z_0, Z_1, Z_2, \ldots, Z_k}, \underbrace{Z_0, Z_1, Z_2, \ldots, Z_k}, \underbrace{Z_0, Z_1, Z_2, \ldots, Z_k}, \ldots\}$$

k = period

Pseudo-random number generator

$$Z_i = (aZ_{i-1} + c) \% m$$

LINEAR CONGRUENTIAL GENERATOR

LEHMER GENERATOR

Z_0 = seed

SAME SEED, SAME SEQUENCE

a, c, and m = carefully chosen constants

$$\{\underbrace{Z_0, Z_1, Z_2, \ldots, Z_k}, \underbrace{Z_0, Z_1, Z_2, \ldots, Z_k}, \underbrace{Z_0, Z_1, Z_2, \ldots, Z_k}, \ldots\}$$

k = period

Pseudo-random number generator

$$Z_i = (aZ_{i-1} + c) \% m$$

LINEAR CONGRUENTIAL GENERATOR

LEHMER GENERATOR

THIS IS VERY BASIC. YOU CAN FIND MUCH BETTER PRNGS OUT THERE.

Z_0 = seed

a, c, and m = carefully chosen constants

$$\{\underbrace{Z_0, Z_1, Z_2, \ldots, Z_k}, \underbrace{Z_0, Z_1, Z_2, \ldots, Z_k}, \underbrace{Z_0, Z_1, Z_2, \ldots, Z_k}, \ldots\}$$

k = period

What is a ***random variate***?

- Bernoulli (discrete)
- Binomial (discrete)
- Geometric (discrete)
- Equilikely (discrete)
- Uniform (continuous)
- Normal or *Gaussian* (continuous)
- Exponential (continuous)

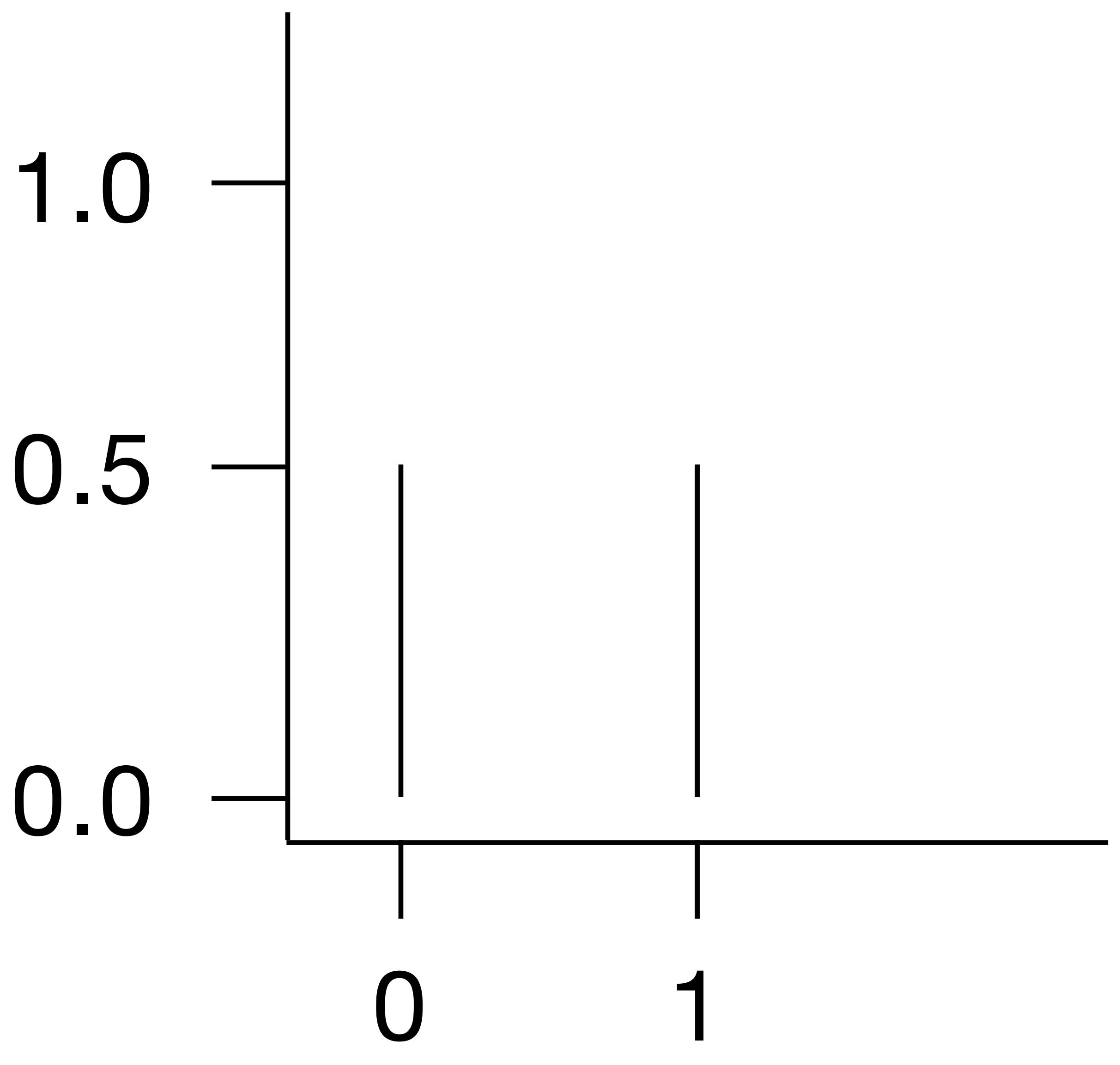
Bernoulli(p)

Two possible outcomes: ***"success"*** (*X=1*) and ***failure*** (*X=0*).

Pr{X=1} = x

Pr{X=0} = 1-x

Range = [0,1]



p=0.8

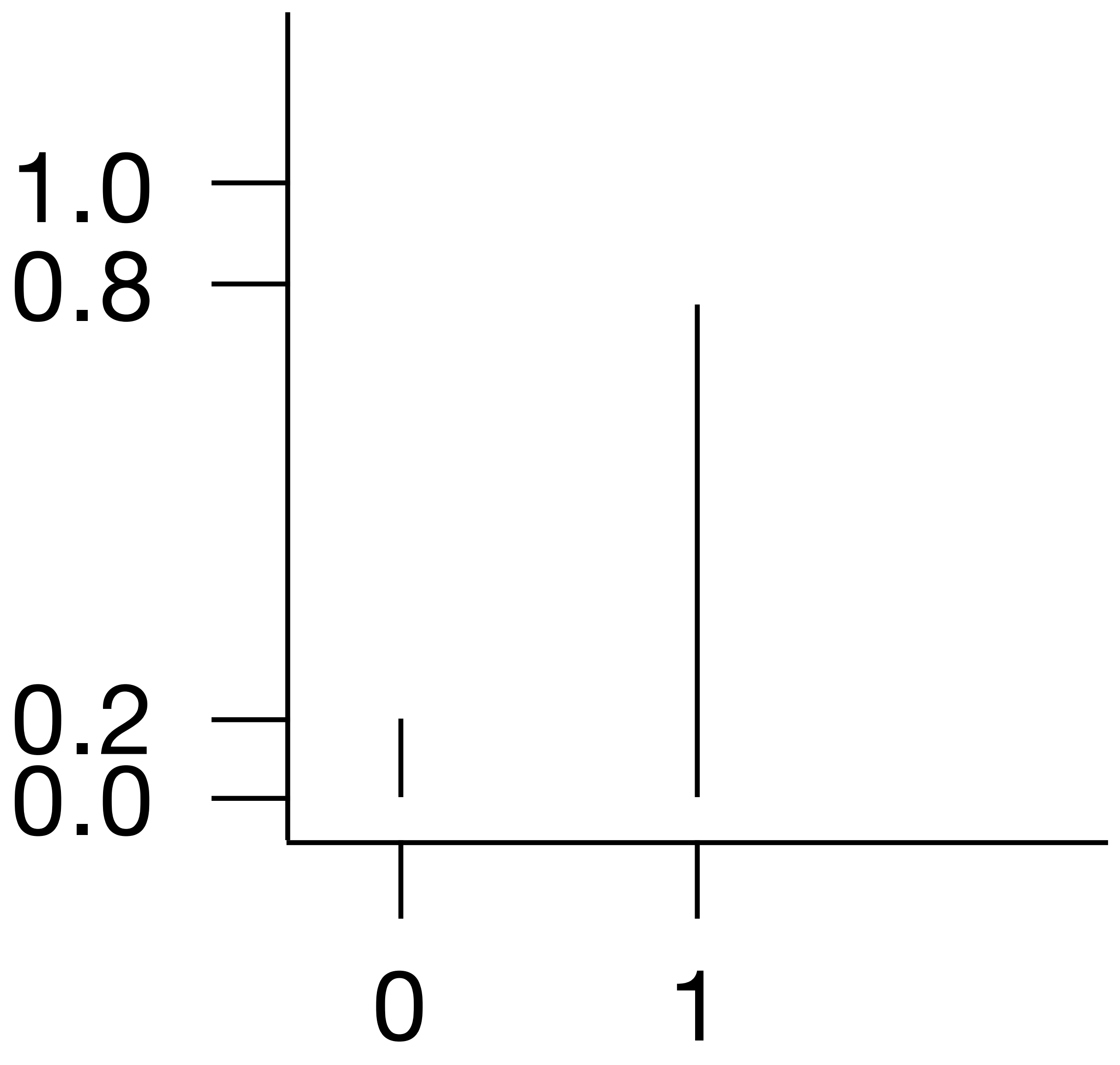
Bernoulli(p)

Two possible outcomes: ***"success"*** ($X=1$) and ***failure*** ($X=0$).

$Pr\{X=1\} = x$

$Pr\{X=0\} = 1-x$

$Range = [0,1]$



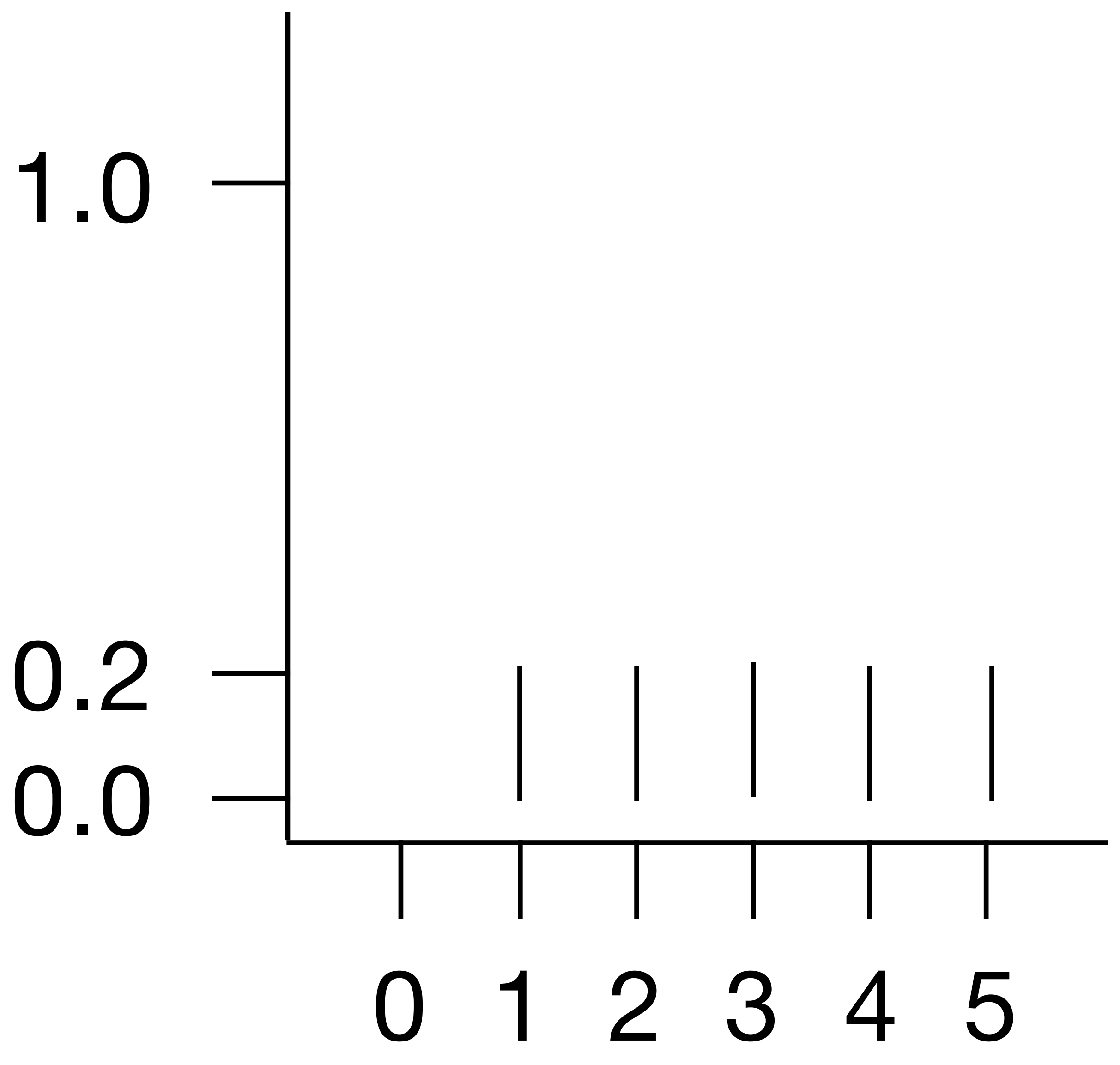
$p=0.8$

Equilikely(a,b)

Possible values are {a, a+1, a+2, …, b}

Range = [a,b]

Pr{X=i} = ?



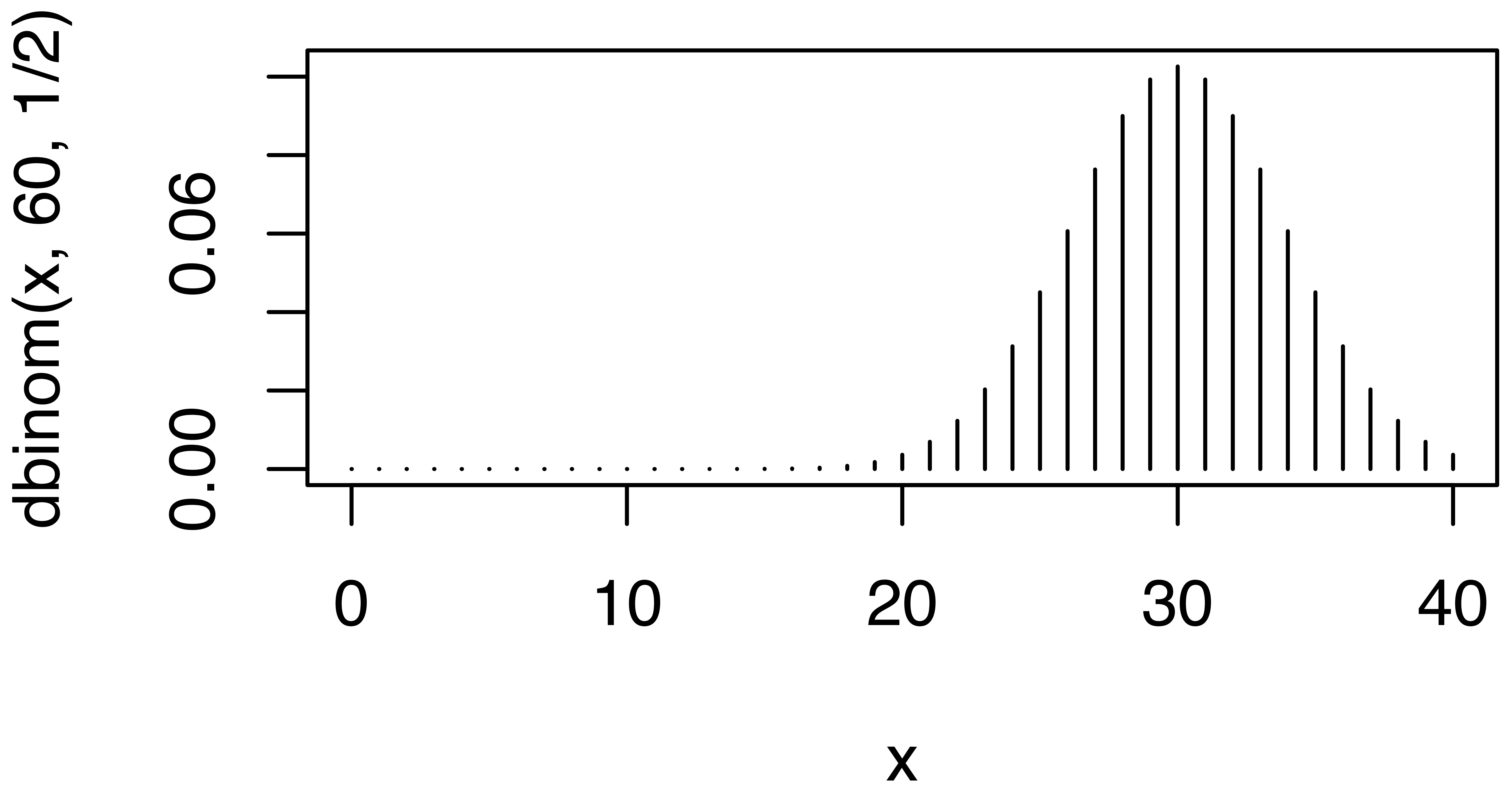
Binomial(n, p)

Repeat a Bernoulli(p) experiment n times and count the number of successes.

What is the range of *Binomial(n,p)*?

$Pr\{X=x\} = ?$

$$\underbrace{SSS\ldots S}_{x}\underbrace{FF\ldots F}_{n-x}$$



Geometric(p)

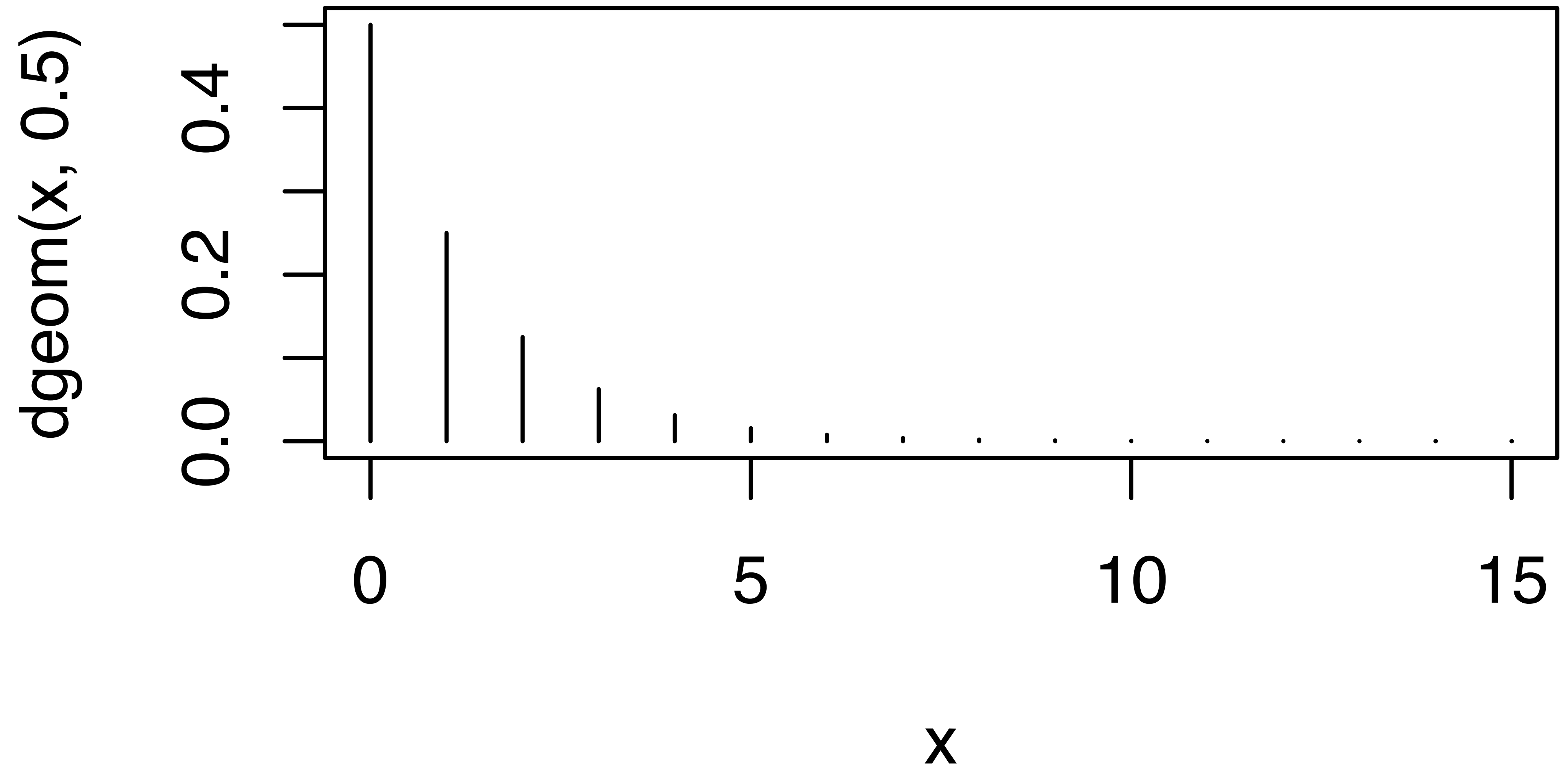
Repeat a Bernoulli(p) experiment until you have a first successes; count the number of failures before you see that success.

What is the range of *Geometric(p)?*

Pr{X=x} = ?

FF… FS

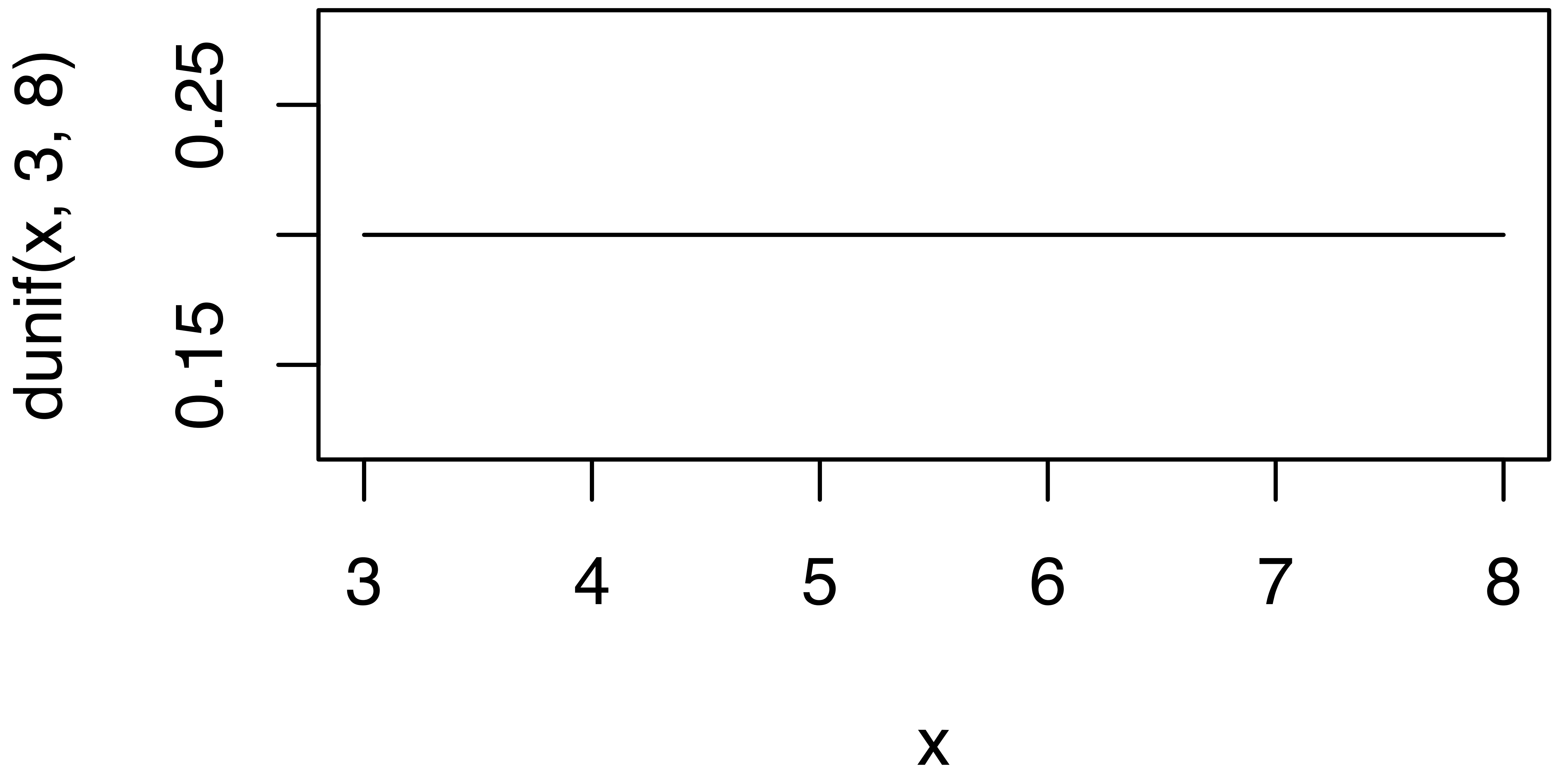
$\underbrace{\text{FF}\ldots\text{ FS}}_{x}$



Uniform(a,b)

a = start

b = end



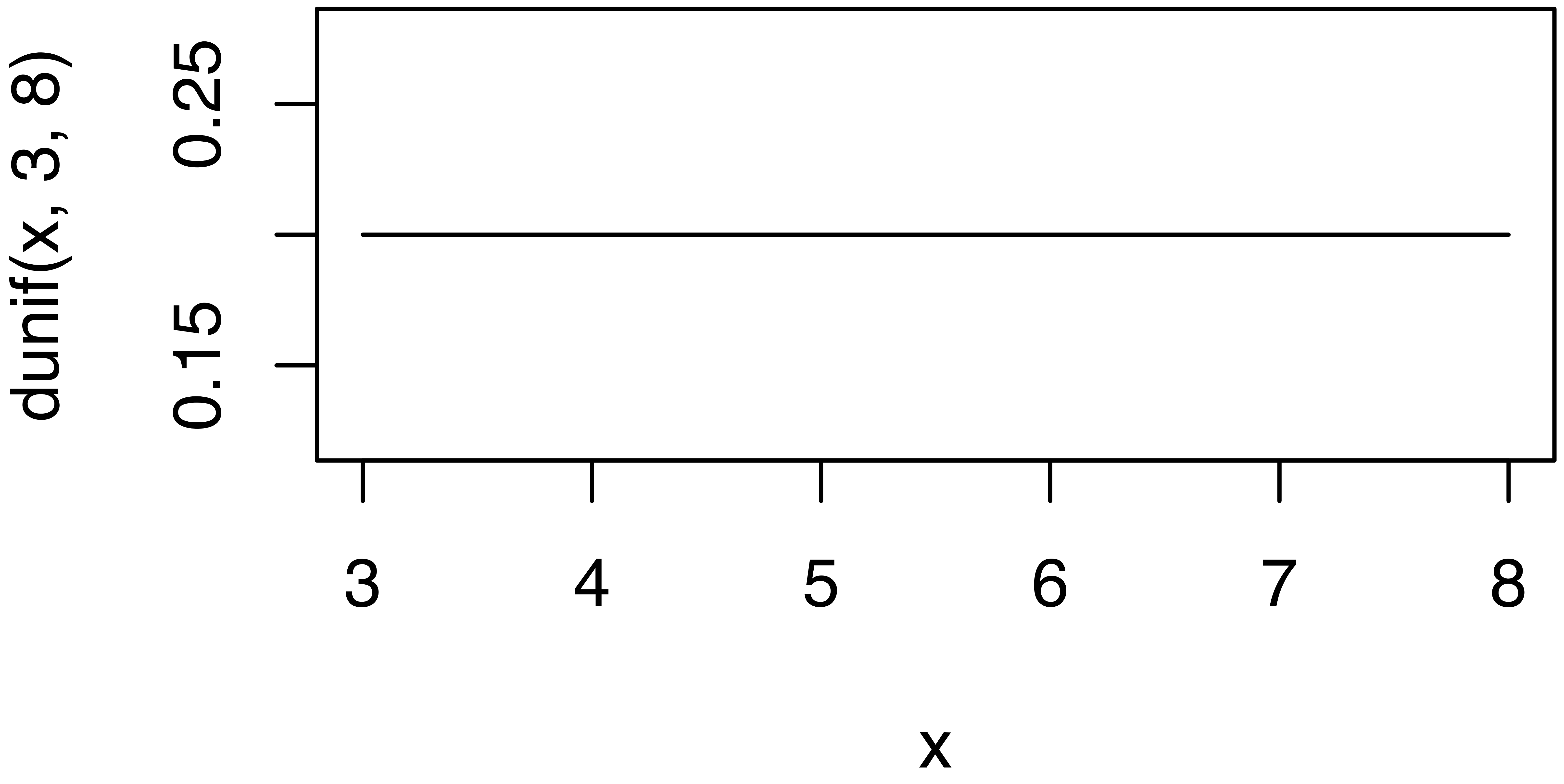
f(x) = probability density function

F(x) = cumulative distribution function

Uniform(a,b)

$$f(x) = \frac{1}{b-a}, \quad a < x < b$$

$$F(x) = \frac{x-a}{b-a}$$

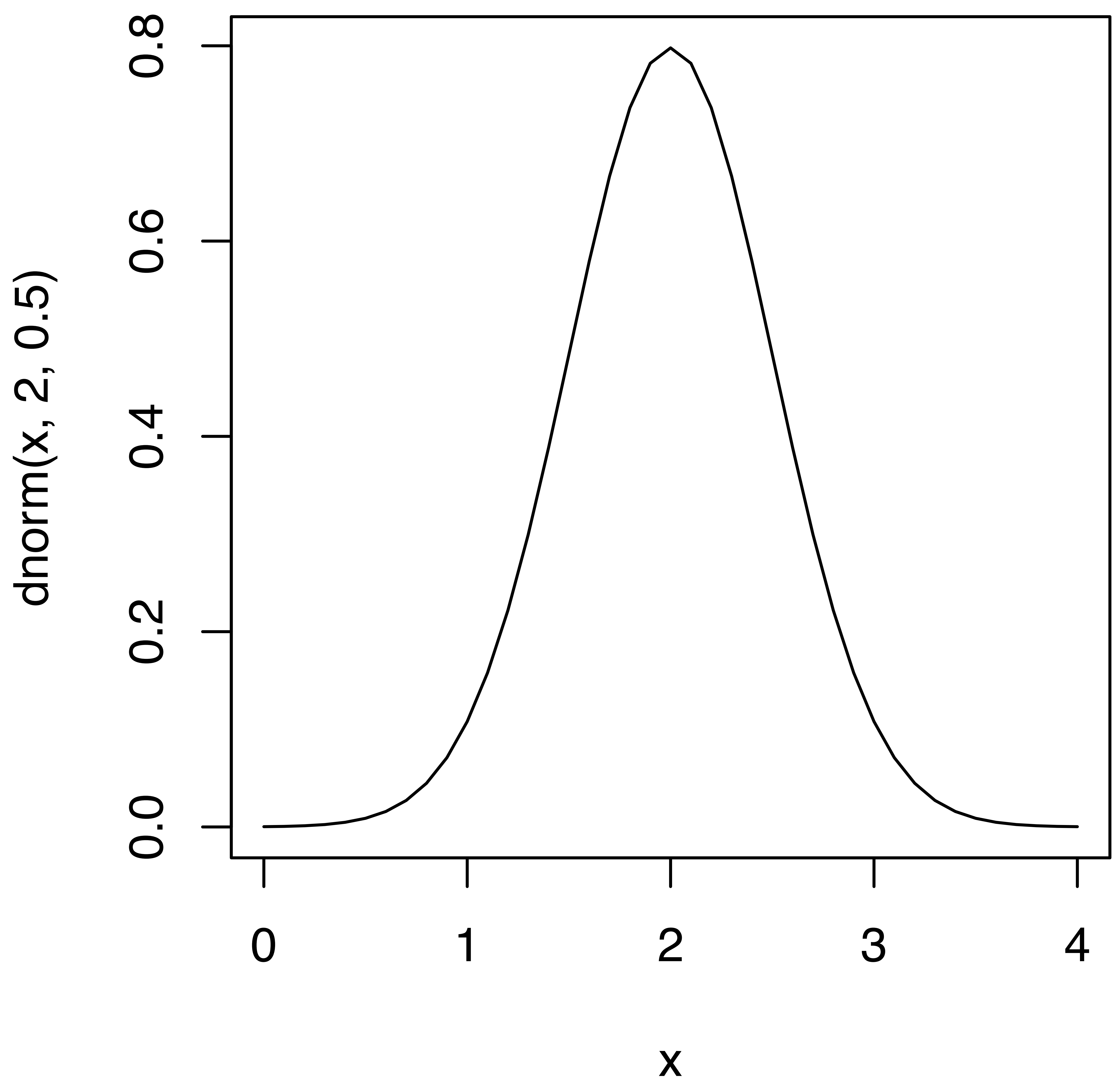


Normal(m,s)

m = mean

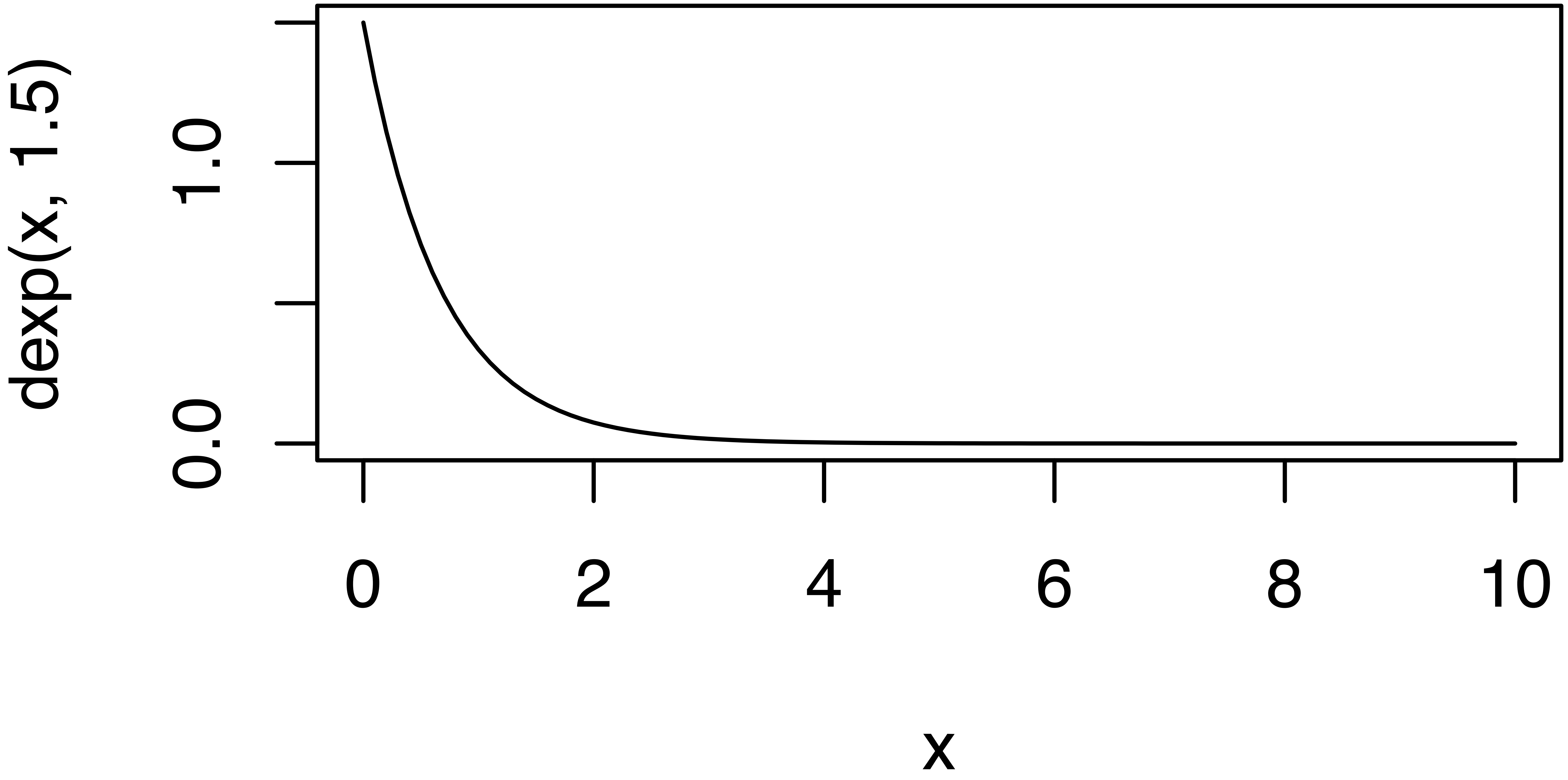
s = standard deviation

What is the range of *Normal(m,s)*?

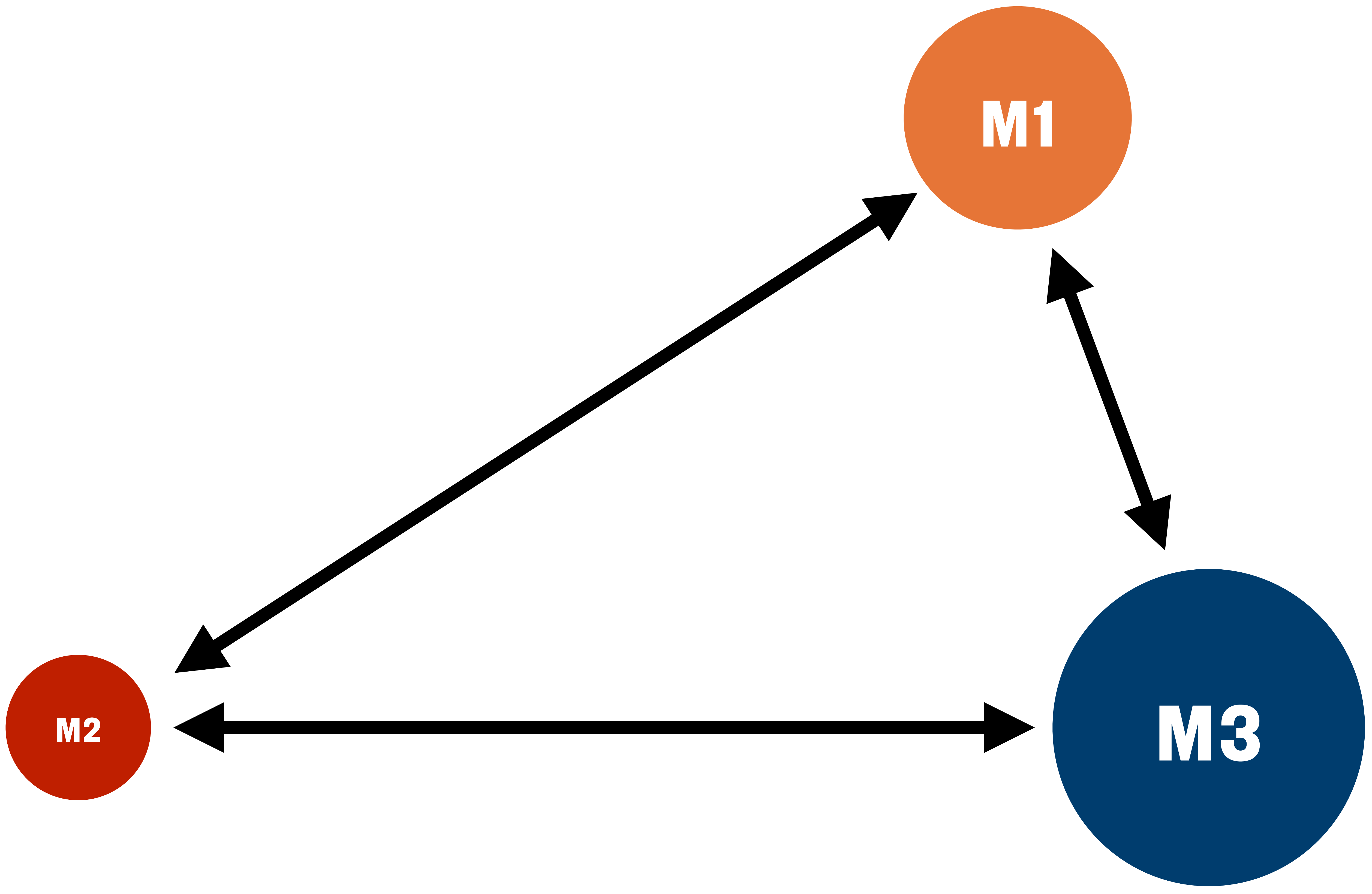


Exponential(r)

r = rate

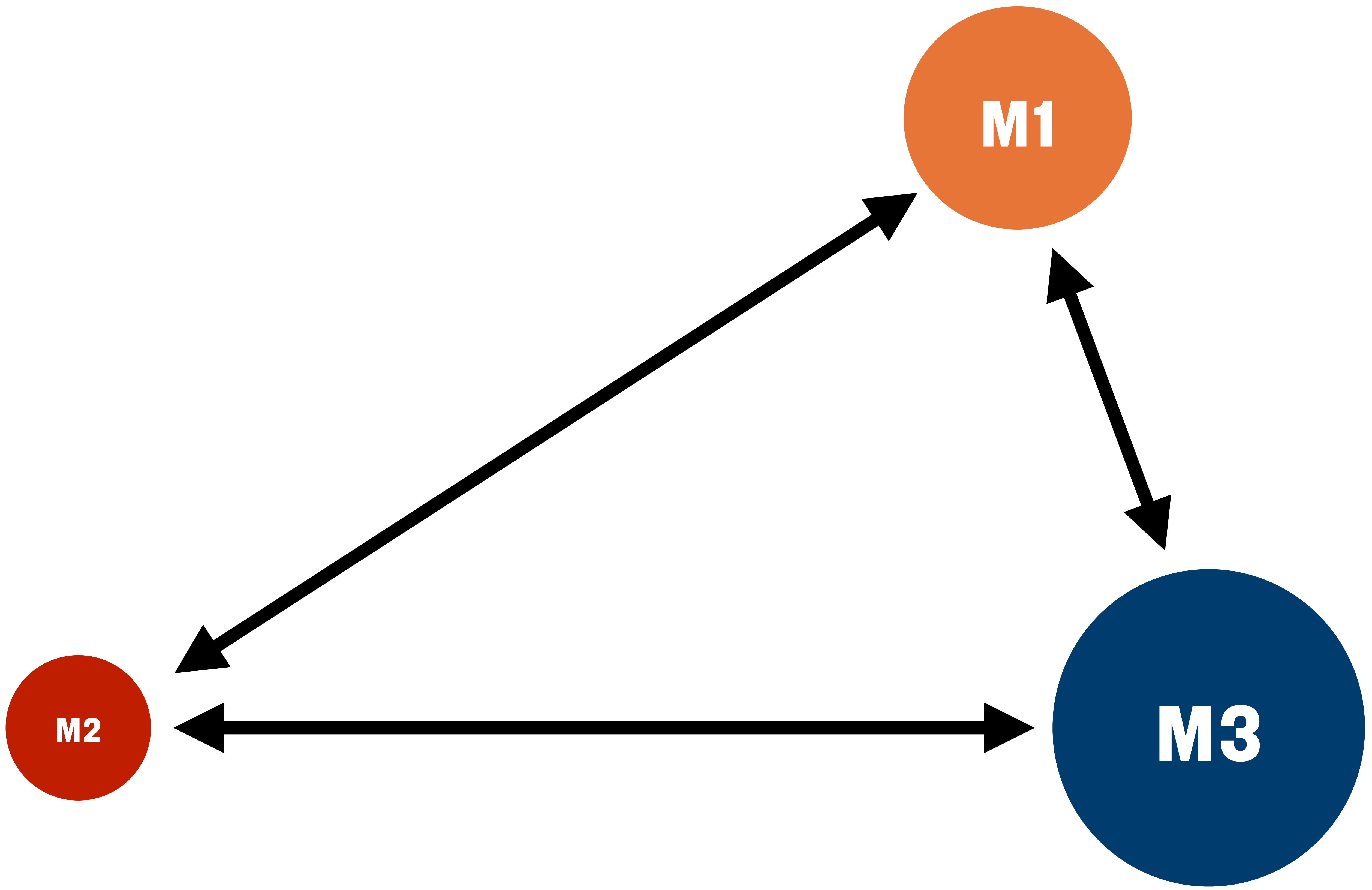


The Tree-body Problem



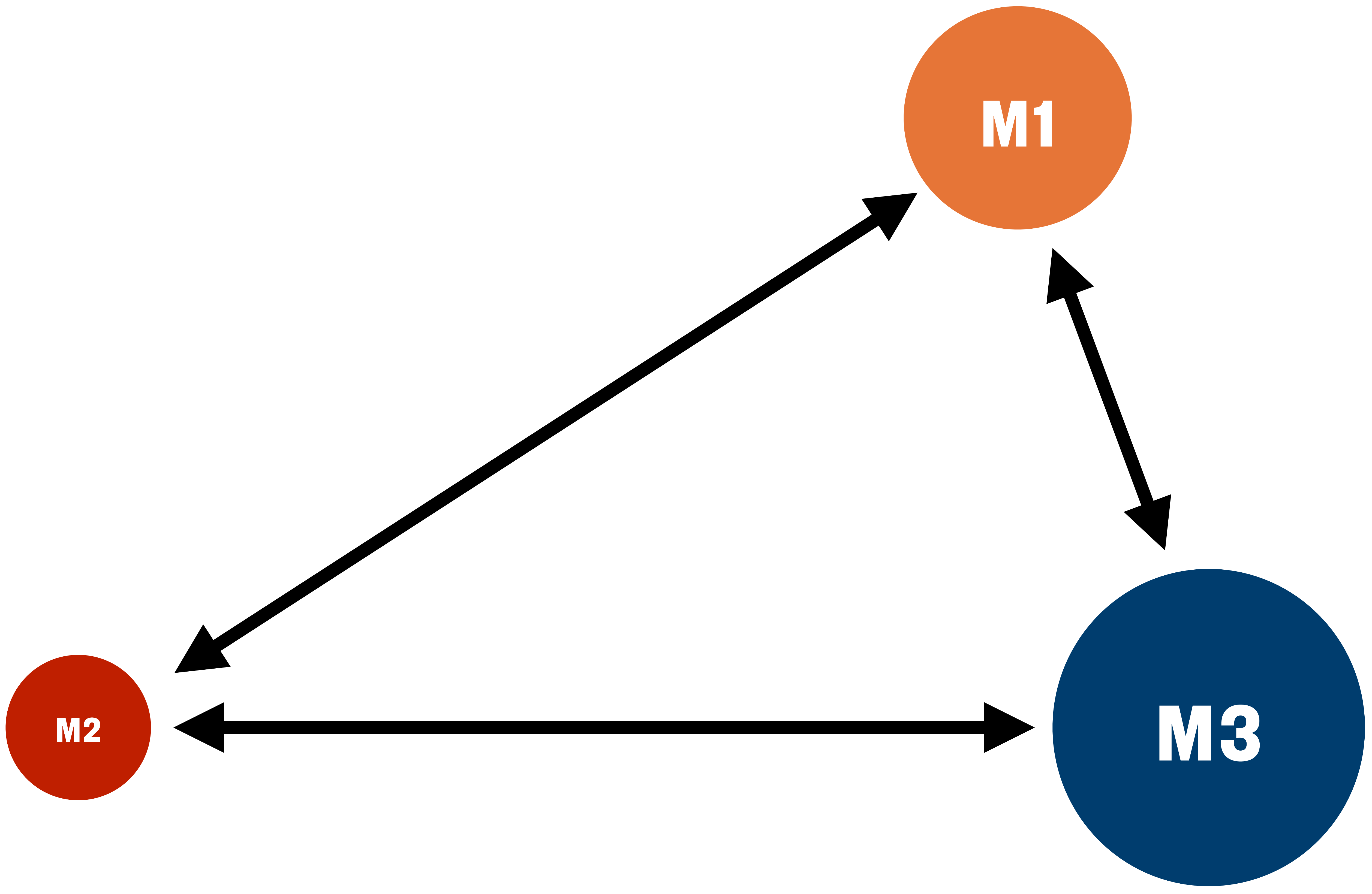
The Tree-body Problem

What do you gain by simulating this?

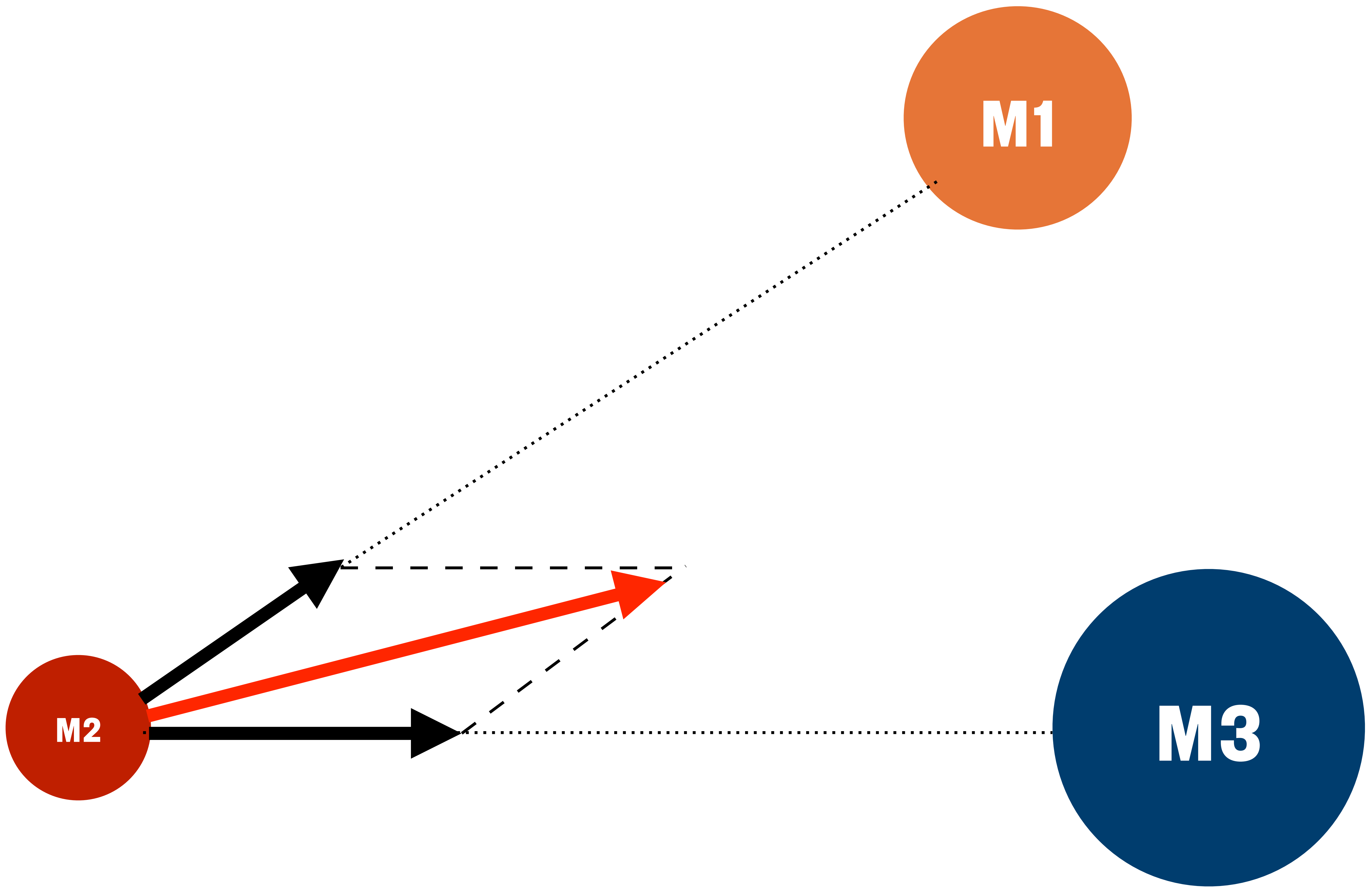


The Tree-body Problem

What does the model need to capture?

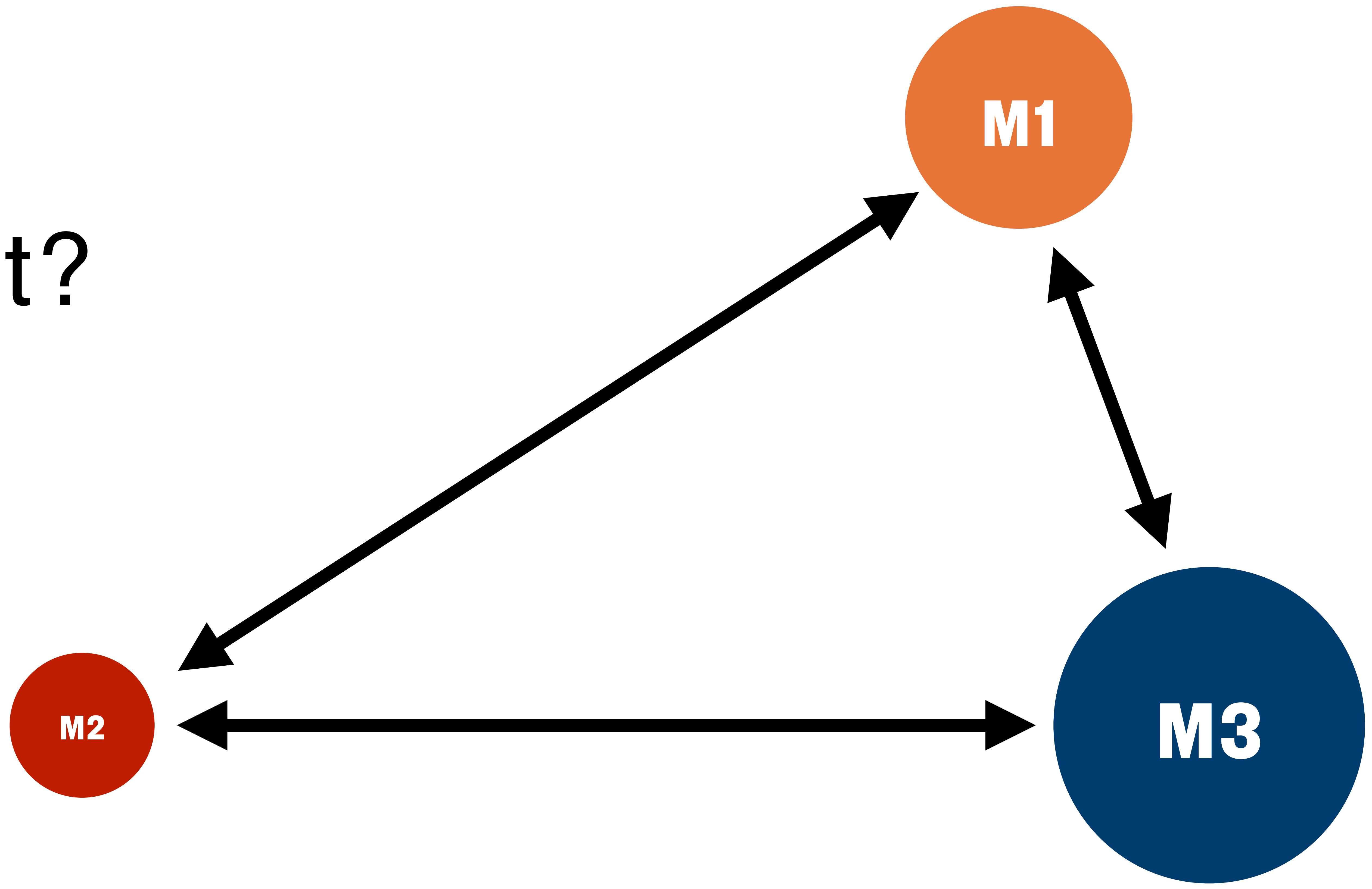


The Tree-body Problem



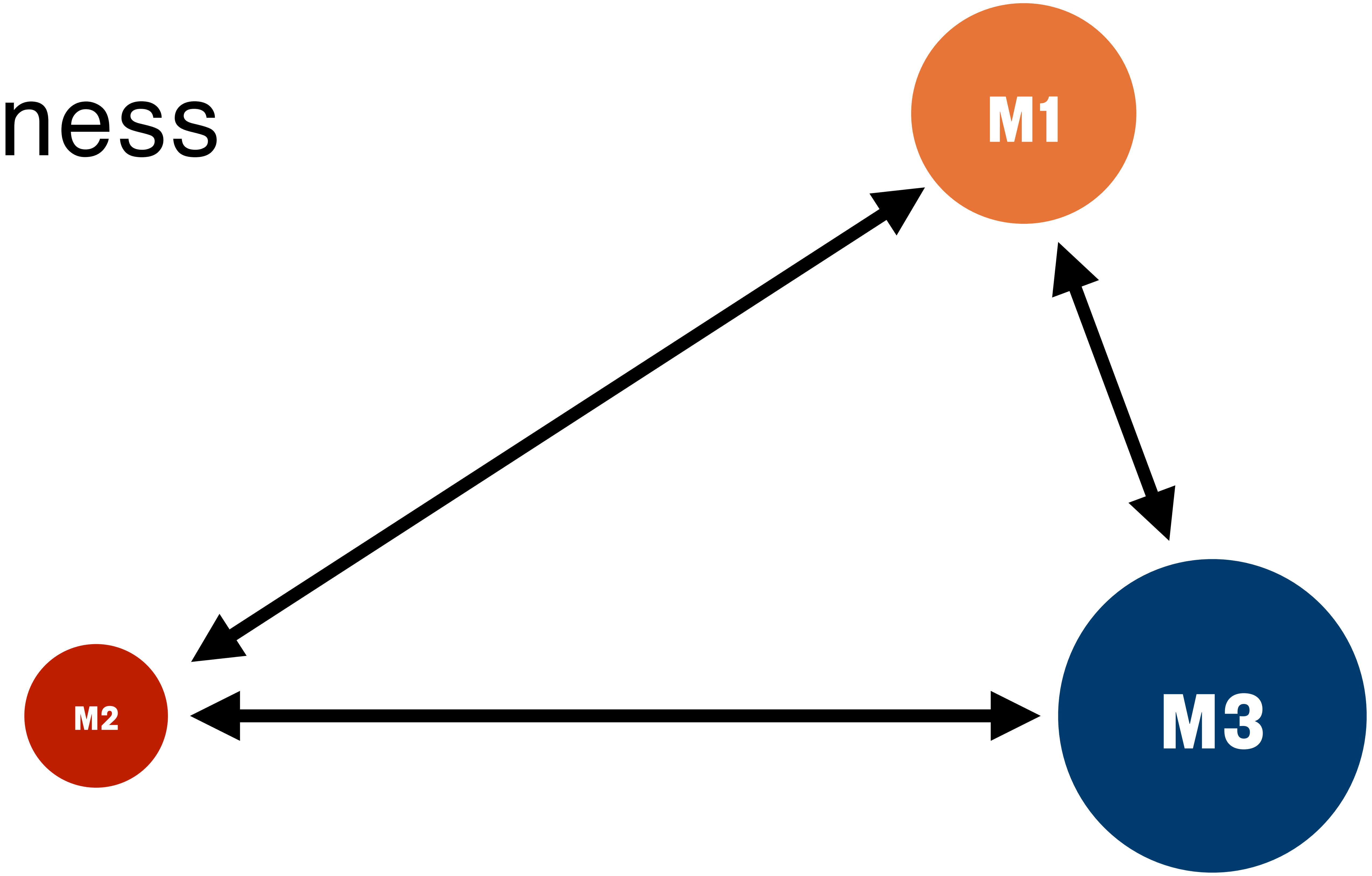
The Tree-body Problem

What defines each simulation experiment?



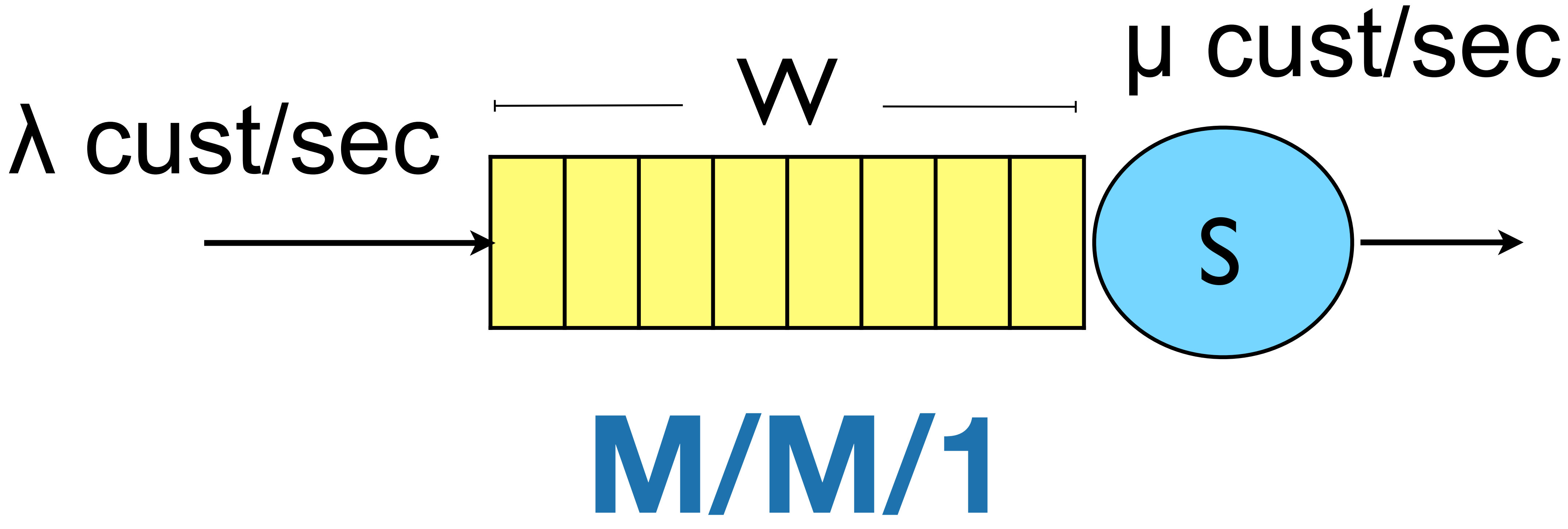
The Tree-body Problem

Where is the randomness in this model?



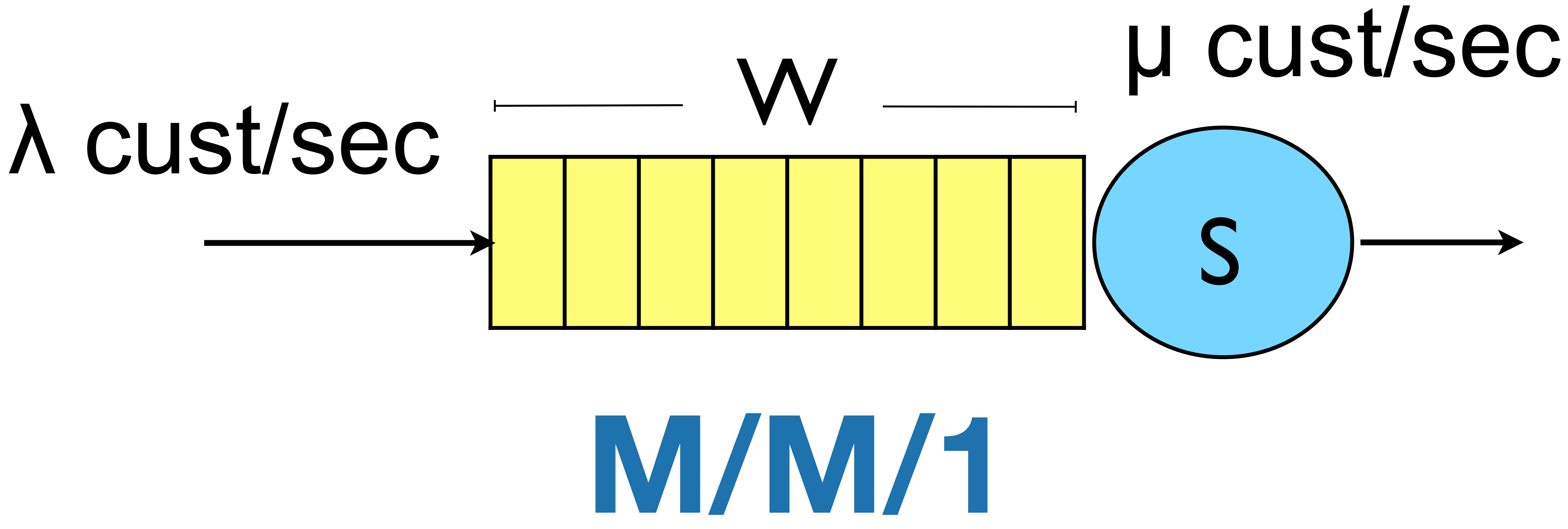
The M/M/1 Queue

What in the world is captured in this abstraction?



The M/M/1 Queue

What do you gain by simulating this?

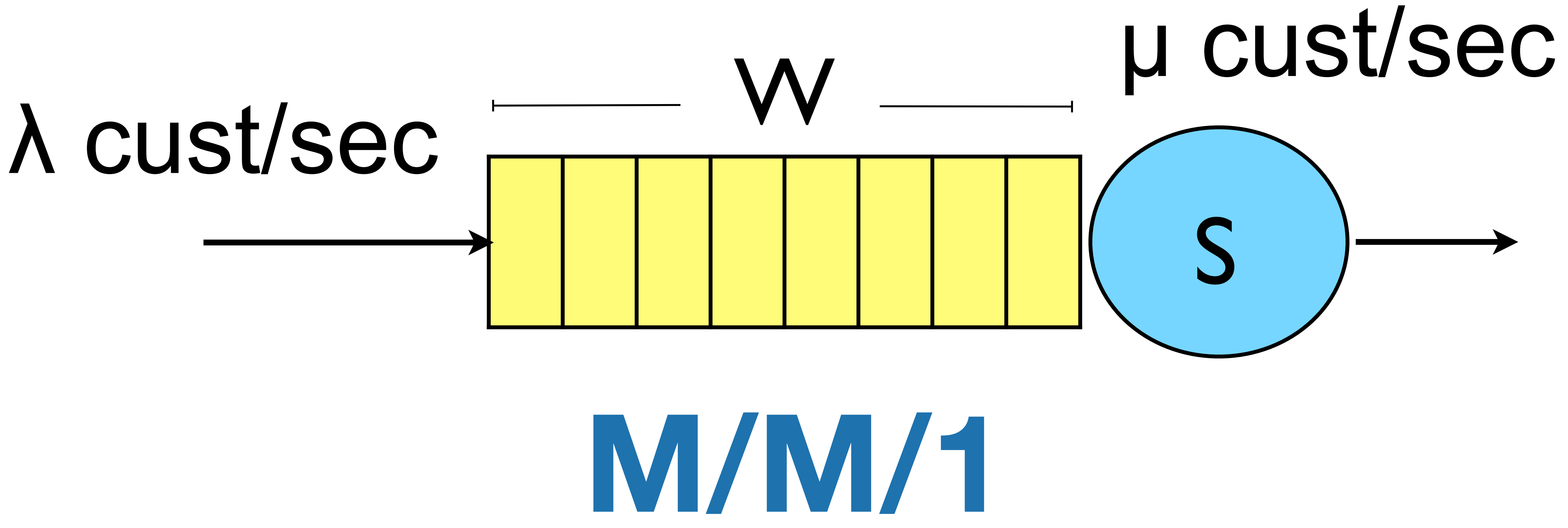


The M/M/1 Queue

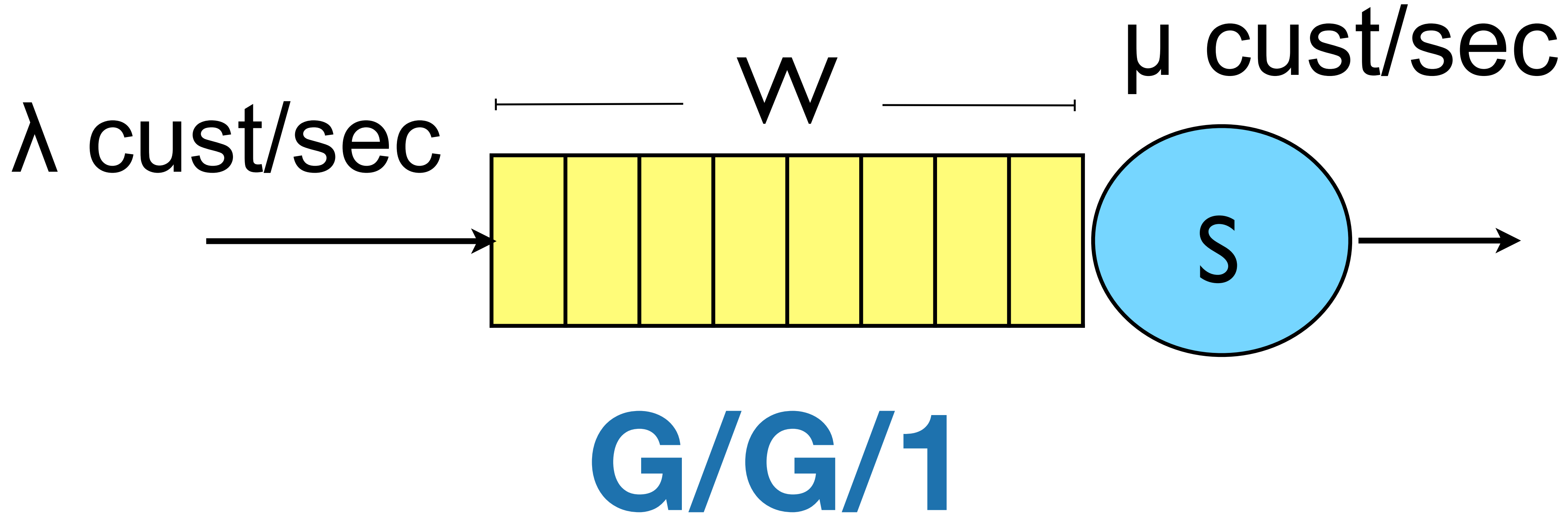
What do you gain by simulating this?

$$L = \frac{\frac{\lambda}{\mu}}{1 - \frac{\lambda}{\mu}}$$

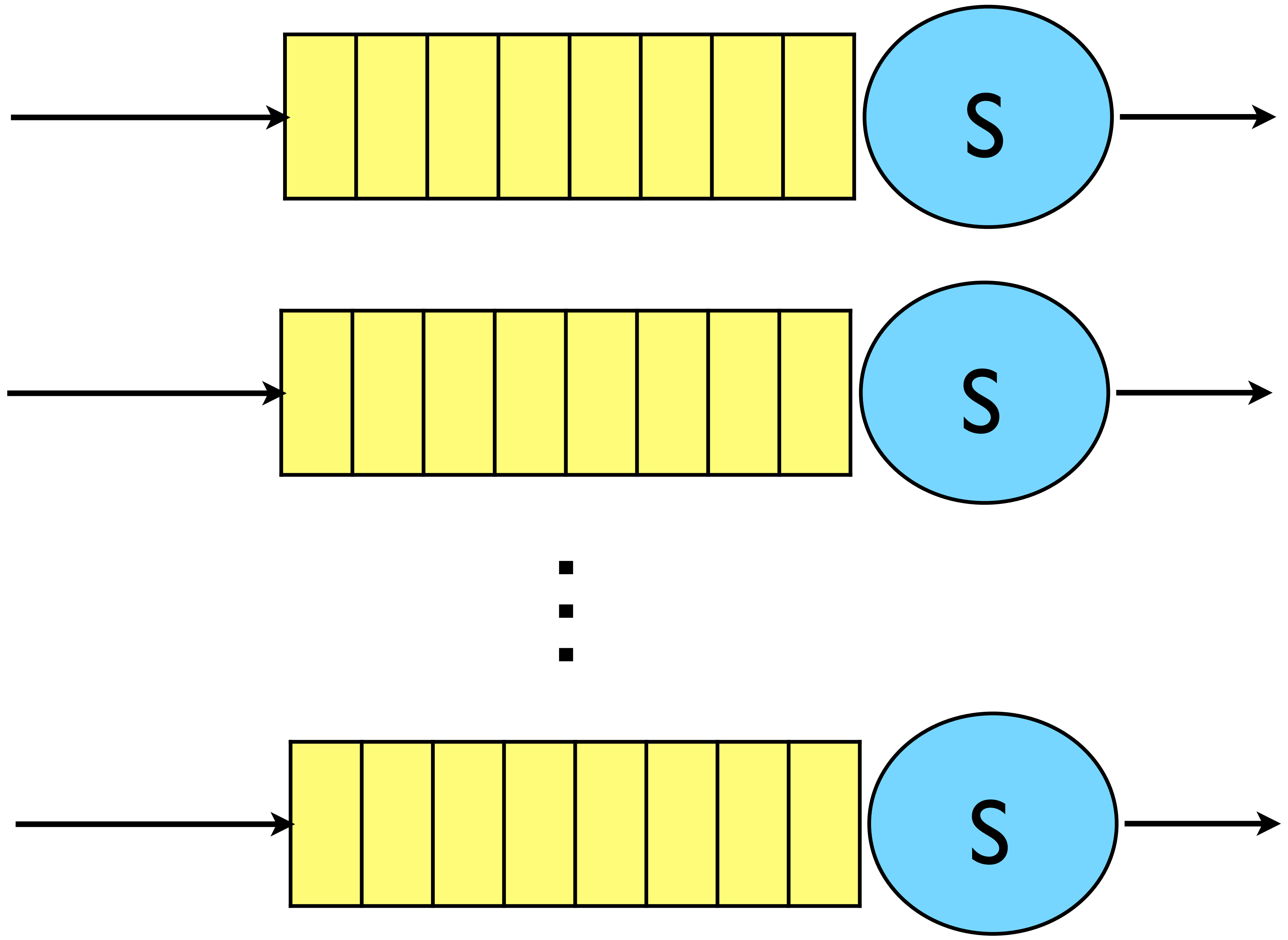
$$W = \frac{1}{\mu - \lambda}$$



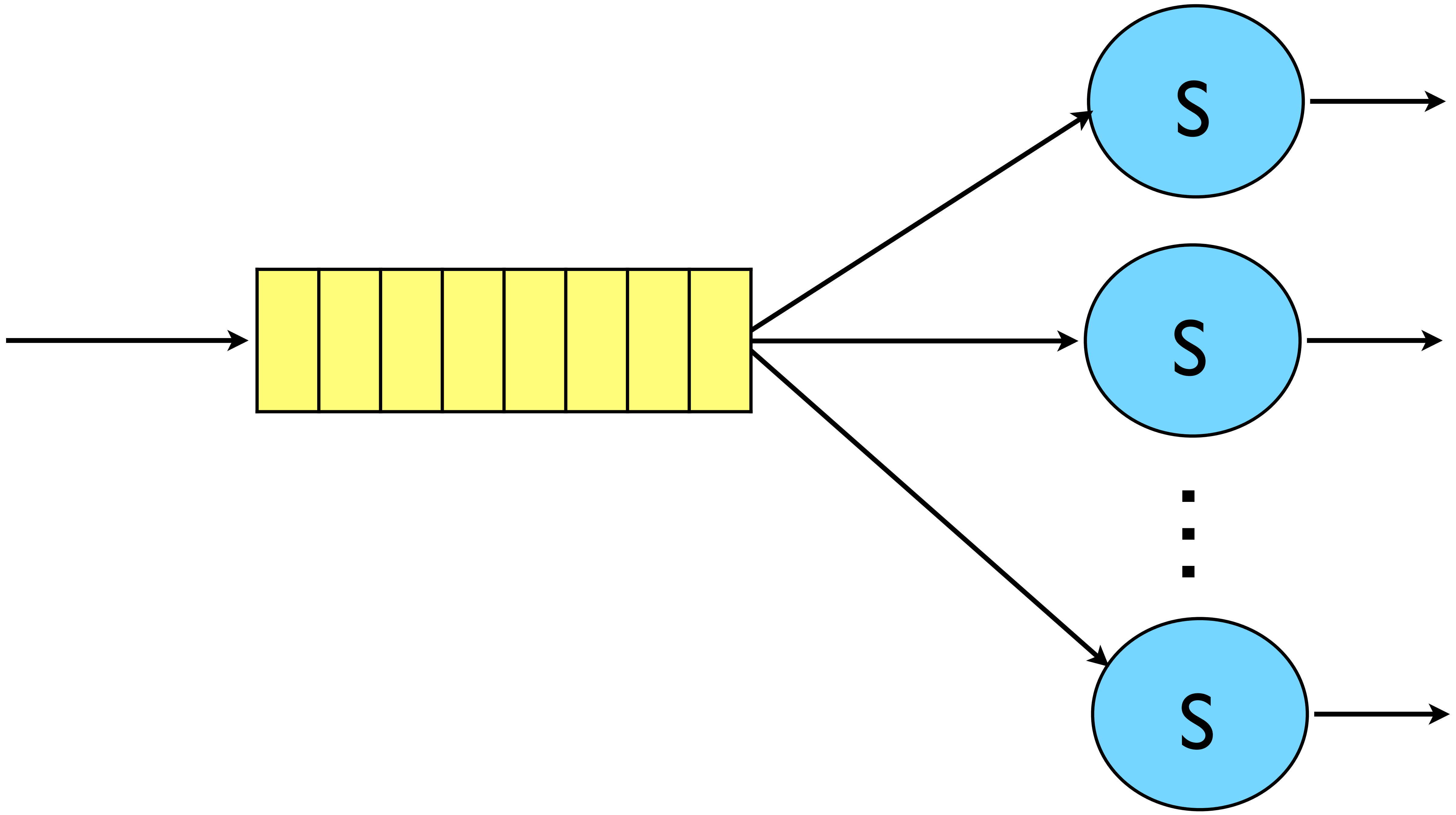
Related Models



Related Models



Related Models



Aircraft Boarding

Aircraft Boarding

What do you gain by simulating this?

Aircraft Boarding

Where is the randomness in this model?

Aircraft Boarding

What do you learn from each simulation experiment?